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Technical Note

Improvement of one-equation boundary-layer integral method for pool film-boiling heat transfer from a downward-facing surface

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1. Introduction

In order to predict quenching processes of ferrous materials or mechanical characteristics of superconductor magnets at quench, better understanding of the mechanism of film boiling heat transfer and establishment of its theoretical model are necessary. In general, film boiling heat transfer is significantly influenced by shapes and orientation of heating surfaces, but, in the case of a horizontal surface facing downward, the heat transfer coefficient becomes the worst. It is, therefore, important to thoroughly clarify the heat transfer characteristics of film boiling heat transfer from a horizontal surface facing downward. So far, a number of experimental studies [1–3,5,6] have been made on this subject, the reported experimental data differ greatly among the researchers. Recently, Nishio et al. [5] indicated that the heat transfer coefficient is enhanced remarkably by disturbances caused by departure bubbles at the outer edge of a heating surface and obtained experimental data of the heat transfer coefficient for pool film boiling with the smooth liquid–vapor interface. Tokita and Djati [6] showed that the heat transfer characteristics change systematically with the height of a heating surface extruded from a circumferential, insulated surface and that there exists a height where the heat transfer coefficient becomes the smallest. They found that this height was 1 mm and the most stable film boiling was realized in this case. On the other hand, a few theoretical studies of pool film boiling heat transfer have also been performed mainly using boundary-layer integral methods. Barron-Dergham [3] and Nishio et al. [5] developed one-equation boundary-layer integral method, where only the vapor film thickness δ is treated as unknown, while Shigechi et al. [4] proposed two-equation boundary layer integral method and derived the governing equations for the vapor film thickness δ and representative radial velocity u_x. These authors assumed linear or quadratic equations to describe the dimensionless temperature profile of the vapor film, i.e., $\theta(\eta) = 1 - \eta$ and $(1 - \eta)^2$, which, however, violate the energy balance at the vapor–liquid interface and result in an improper dependence of Nusselt number on the superheat.

The purpose of the present note is to improve existing one-equation boundary-layer integral method for film boiling heat transfer from a downward-facing surface. To this end, first, the dimensionless temperature profile of a vapor film is determined in order to simultaneously satisfy the energy balance at the liquid–vapor interface and the energy equation. With the dimensionless temperature profile thus determined, the governing equation for the dimensionless vapor film thickness δ is derived and solved numerically. Then, an analytical formula for the heat transfer characteristics is established based on the numerical results for δ. Finally, the validity of the obtained heat transfer correlation is addressed in comparison with available experimental data.

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Nomenclature

a_n	expansion coefficient
C_0	coefficient
C_P	specific heat at constant pressure
D	diameter of a heating surface
Gr	Grashof number $(= g(\rho_L/\rho_V - 1)D^3/\nu_V^2)$
g	gravitational acceleration
$\bar{h}$	mean convective heat transfer coefficient
K	dimensionless quantity defined by $(1 + S_P/2)/S_P$
K^*	quantity defined by $1/(3 + 2a_1)$
k_V	thermal conductivity of vapor
l	latent heat of evaporation
N	maximum order of a power series
Nu	mean convective Nusselt number $(= \bar{h}D/k_V)$
P	pressure
Pr_V	Prandtl number of vapor $(= C_{PV}\mu_V/k_V)$
$\bar{q}$	mean convective heat flux
q_R	radiative heat flux
S	radius at the outer edge of a vapor film
S_P	dimensionless superheat $(= C_{PV}\Delta T_{sat}/\ell)$
T	temperature
T_w	temperature of a heating surface
T_{sat}	saturation temperature
ΔT_{sat}	the degree of superheat $(= T_w - T_s)$
u	velocity component in the radial direction
u_x	representative radial velocity
w	velocity component in the vertical direction
w_S	vertical velocity component of vapor at the liquid–vapor interface
X	film-boiling Rayleigh number $(= Gr\, Pr_V K)$
x	radial coordinate
y	coordinate normal to a heating surface
α	absorptivity
β_2, β_3	constants
γ_3, γ_4	constants
δ	thickness of a vapor film
δ_0	thickness of a vapor film at the center of a heating surface
η	dimensionless vertical coordinate $(= y/\delta)$
ε	emissivity
θ	dimensionless temperature $(= (T - T_s)/\Delta T_{sat})$
μ_V	viscosity of vapor
ν_V	kinematic viscosity of vapor
ϕ	dimensionless thickness of vapor film $(= (\delta/D)^4(Gr\, Pr_V K^*/3S_P)^{4/5})$
ρ	density
ψ	dimensionless radial velocity

Subscripts

L	liquid
V	vapor

2. Theoretical analysis

In order to perform the analysis, we assume that: (1) a stable vapor film is formed on the lower surface of a horizontal circular plate of diameter D and of uniform temperature T_w; (2) the liquid–vapor interface is smooth and the effect of surface tension can be disregarded; (3) the boundary-layer approximation is valid; (4) the flow field of vapor is viscous; (5) the temperature at the liquid–vapor interface is equal to the saturation temperature T_S; (6) the radial velocity of vapor at the liquid–vapor interface is zero; (7) the physical properties of vapor are constant, and they are estimated at the film temperature; and (8) the contribution of radiation heat transfer is negligible.

Under the foregoing assumptions, the conservation laws of mass, momentum and energy are mathematically described as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} + \frac{u}{x} = 0, \tag{1}$$

or

$$w_S x = -\frac{\partial}{\partial x}\int_0^{\delta} ux\,\mathrm{d}y. \tag{2}$$

$$0 = -\frac{\partial P_V}{\partial x} + \mu_V \frac{\partial^2 u}{\partial y^2} \tag{3}$$

$$0 = \rho_V g - \frac{\partial P_V}{\partial y} \tag{4}$$

$$\rho_V C_{PV}\left(u\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial y}\right) = k_V \frac{\partial^2 T}{\partial y^2} \tag{5}$$

The boundary conditions for these equations are

$$y = 0: \qquad u = w = 0, \quad T = T_W \tag{6}$$

$$y = \delta: \qquad u = 0, \tag{7}$$

$$P_V = P_L, \tag{8}$$

$$T = T_S, \tag{9}$$

$$-k_V \frac{\partial T}{\partial y}|_\delta = -l\rho_V w_S, \tag{10}$$

$$x = 0: \quad u = 0, \quad \delta = \delta_0 \tag{11}$$

$$x = S\left(> \frac{D}{2}\right): \quad \delta = 0, \quad \frac{d\delta}{dx} = -\infty \tag{12}$$

We assumed $S = 1.01(D/2)$ in accord with Nishio et al. [5].

The radial pressure gradient of vapor [3–5] in Eq. (3) may be written as

$$\frac{\partial P_V}{\partial x} = (\rho_L - \rho_V)g\frac{d\delta}{dx}. \tag{13}$$

Substituting Eq. (13) into Eq. (3) and solving it analytically yield the following results:

$$u = u_x\psi(\eta) \tag{14}$$

with

$$u_x = -\left[\frac{g(\rho_L - \rho_V)}{2\mu_V}\right]\delta^2\frac{d\delta}{dx} \tag{15}$$

and

$$\psi(\eta) = \eta - \eta^2. \tag{16}$$

Next, we rewrite the governing equations in boundary-layer integral forms.

Substituting Eq. (2) into Eq. (10) results in

$$\frac{d}{dx}\left[\int_0^\delta ux\, dy\right] + \frac{xk_V}{\rho_V l}\frac{\partial T}{\partial y}|_\delta = 0, \tag{17}$$

Integrating Eq. (5) with respect to y, we can obtain

$$\rho_V C_{PV}\frac{d}{dx}\left[x\int_0^\delta u(T - T_S)\, dy\right] = k_V x\left(\frac{\partial T}{\partial y}|_\delta - \frac{\partial T}{\partial y}|_0\right), \tag{18}$$

Here, we assume that the temperature distribution can be represented as

$$T = T_S + \Delta T_{sat}\theta(\eta). \tag{19}$$

Introducing Eqs. (14) and (19) into Eqs. (17) and (18), we can obtain the following differential equations with respect to the vapor thickness δ:

$$\beta_2\frac{d}{dx}(x\delta u_x) + \frac{k_V\Delta T_{sat}\gamma_4}{\rho_V l}\frac{x}{\delta} = 0, \tag{20}$$

$$\rho_V C_{PV}\beta_3\frac{d}{dx}(x\delta u_x) - k_V(\gamma_4 - \gamma_3)\frac{x}{\delta} = 0, \tag{21}$$

where β_2, β_3 γ_3 and γ_4 are defined as

$$\beta_2 = \int_0^1 \psi\, d\eta\left(= \frac{1}{6}\right), \quad \beta_3 = \int_0^1 \psi\theta\, d\eta$$

$$\gamma_3 = \frac{d\theta}{d\eta}|_0, \quad \gamma_4 = \frac{d\theta}{d\eta}|_1 \tag{22}$$

Eliminating $\frac{d}{dx}(x\delta u_x)$ from Eqs. (20) and (21) results in

$$\frac{S_P}{\beta_2}\left(\beta_3 + \frac{\beta_2}{S_P}\right) = \frac{\gamma_3}{\gamma_4}, \tag{23}$$

where S_P denotes the dimensionless superheat defined by $C_{PV}\Delta T_{sat}/l$. In addition to the boundary conditions (6) and (9), i.e., $\theta(0) = 1$ and $\theta(1) = 0$, $\theta(\eta)$ must satisfy Eq. (23). Moreover, since the energy equation (5) may be rewritten in form of $w = (\frac{k_V}{\rho_V C_{PV}}\frac{\partial^2 T}{\partial y^2} - u\frac{\partial T}{\partial x})/\frac{\partial T}{\partial y}$ and the velocity components u and w must be zero at $y = 0$, the following relation should hold:

$$w(0) = \left(k_V/\rho_V C_{PV}\right)\frac{\partial^2 T}{\partial y^2}\bigg|_0 \bigg/ \frac{\partial T}{\partial y}\bigg|_0 = 0 \tag{24}$$

When θ is expanded into a power series of η, i.e., $\theta = \sum_{n=0}^N a_n\eta^n$, Eq. (24) postulates that $a_1 \neq 0$ and $a_2 = 0$. Although a value of N in the series expansion of θ should be equal to or greater than 3, there exist only four constraints on $\theta(\eta)$ and thus N must be 3:

$$\theta(\eta) = a_0 + a_1\eta + a_3\eta^3 \tag{25}$$

Substituting the boundary conditions (6) and (9) into Eq. (25), we have

$$a_0 = 1 \quad \text{and} \quad a_3 = -(1 + a_1). \tag{26}$$

Using Eq. (23) together with Eqs. (22) and (26), we can determine a_1 as follows:

$$a_1 = -5\left(\frac{5}{12} + \frac{1}{2S_P}\right) + 5\sqrt{\frac{49}{3600} + \frac{13}{60S_P} + \frac{1}{4S_P^2}}. \tag{27}$$

This indicates that $\theta(\eta)$ involves the dimensionless superheat S_P as a parameter. Note that, for $S_P \to \infty$ $(\Delta T_{sat} \to \infty)$, a_1 reduces to -1.5. Under the condition of Eq. (23), Eqs. (20) and (21) are equivalent, meaning that only either of them should be solved. In the present study, we adopted Eq. (20) as the governing equation for δ. Using Eq. (15), we can rewrite Eq. (20) as follows:

$$\frac{d}{dx}\left(x\frac{d\delta^4}{dx}\right) - \frac{8k_V\mu_V\Delta T_{sat}\gamma_4}{\rho_V lg(\rho_L - \rho_V)\beta_2}\frac{x}{\delta} = 0, \tag{28}$$

The boundary conditions for Eq. (28) are

$$x = 0: \qquad \delta = \delta_0, \quad \frac{d\delta}{dx} = 0, \tag{29}$$

$$x = S: \qquad \delta = 0, \quad \frac{d\delta}{dx} = -\infty. \tag{30}$$

To rewrite Eqs. (28)–(30) into dimensionless forms, the following quantities are introduced:

$$G_r = g(\rho_L/\rho_V - 1)D^3/\nu_V^2, \quad K^* = 1/(3 + 2a_1),$$
$$Pr_V = C_{PV}\mu_V/k_V, \quad \eta = r/(D/2)$$

$$\phi = (\delta/D)^4(Gr\, Pr_V K^*/3S_P)^{4/5},$$
$$\phi_0 = (\delta_0/D)^4(Gr\, Pr_V K^*/3S_P)^{4/5} \tag{31}$$

Introducing these quantities into Eq. (28) yields

$$\frac{d}{d\eta}\left(\eta\frac{d\phi}{d\eta}\right) + \frac{4\eta}{\phi^{1/4}} = 0. \tag{32}$$

The boundary conditions are also rewritten as

$$\eta = 0: \qquad \phi = \phi_0, \quad \frac{d\phi}{d\eta} = 0, \tag{33}$$

$$\eta = 1.01: \qquad \phi = 0, \quad \frac{d\phi}{d\eta} = -\infty. \tag{34}$$

Eq. (32) was solved numerically using a fourth-order Runge–Kutta method. The procedure is as follows: since ϕ_0 is not known beforehand, we proceed the computation using an arbitrary value of ϕ_0 and then find a correct value of ϕ_0, for which Eq. (34) is fully satisfied. In the computation, the dimensionless radius was equally divided into 100 increments. To check the accuracy, the numerical result for ϕ_0 was compared with a more accurate result, which was obtained by using 200 equally spaced divisions. The comparison showed that both results agreed up to four significant digits and, in consequence, a value of ϕ_0 is 1.0899. Once ϕ or δ are obtained, we can readily evaluate the mean heat flux $\bar{q}$, heat transfer coefficient $\bar{h}$ and Nusselt number Nu from the following relations:

$$\bar{q} = 2\pi\int_0^{D/2}\left(-k_V\frac{\partial T}{\partial y}\Big|_0\right)x\,dx/(\pi D^2/4). \tag{35}$$

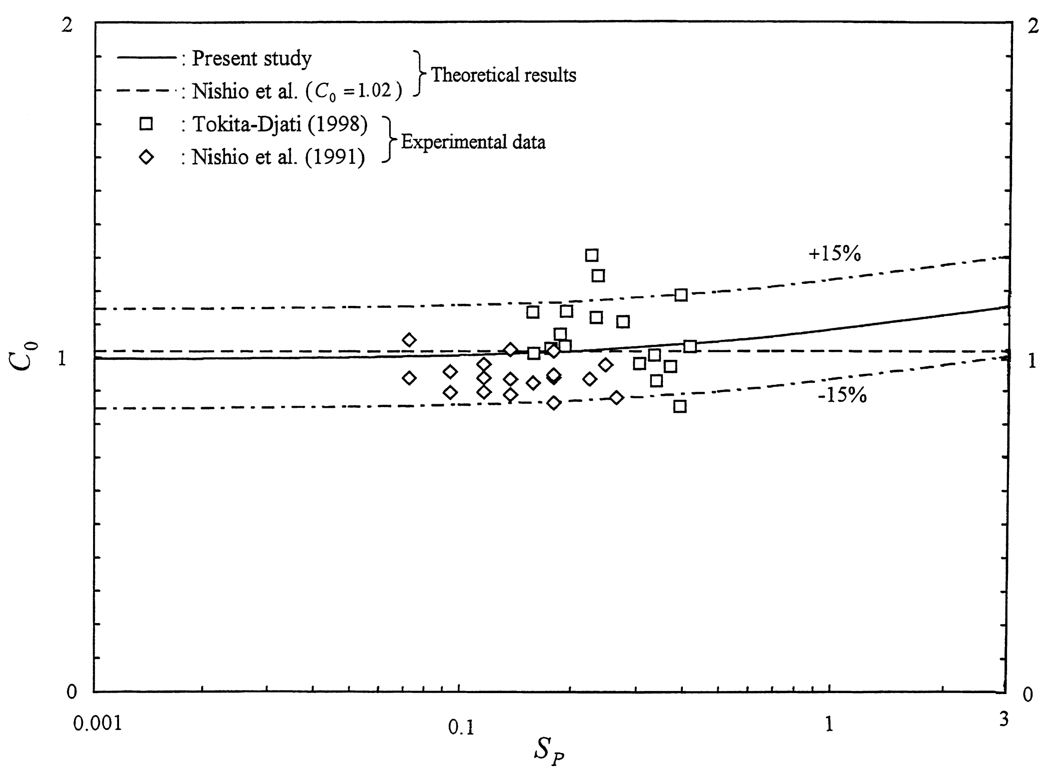

Fig. 1. Correlation between experimental values of $Nu/X^{0.2}$ and theoretical ones computed from Eqs. (37) and (39).

$$\bar{h} = \frac{\bar{q}}{\Delta T_{\text{sat}}} = -\frac{8k_{\text{V}}a_1}{D^2}\int_0^{D/2}\frac{x\,\mathrm{d}x}{\delta}. \tag{36}$$

$$Nu = \frac{\bar{h}D}{k_{\text{V}}} = -\frac{8a_1}{D}\int_0^{D/2}\frac{x\,\mathrm{d}x}{\delta} = C_0X^{1/5}. \tag{37}$$

Here, C_0 is defined as

$$C_0 = -\frac{2a_1}{3^{1/5}}\left[\left(1+\frac{S_{\text{P}}}{2}\right)(3+2a_1)\right]^{-1/5}\int_0^1\frac{\eta}{\phi^{1/4}}\,\mathrm{d}\eta \tag{38}$$

Since a value of the integral defined by $\int_0^1 \frac{\eta}{\phi^{1/4}}\,\mathrm{d}\eta$ was found to be 0.6200, Eq. (38) reduces to

$$C_0 = 0.9954(-a_1)\left[\left(1+\frac{S_{\text{P}}}{2}\right)(3+2a_1)\right]^{-1/5}. \tag{39}$$

When $S_{\text{P}}\to 0$, C_0 is asymptotic to 0.9954: this is very close to the theoretical result obtained by Nishio et al. [5], who assumed a linear temperature profile of vapor film and derived $C_0 = 1.02$ utilizing a variational method.

3. Results and discussion

Correlation between experimental values of $C_0\,(= Nu/X^{0.2})$ and theoretical ones estimated from Eqs. (37) and (39) is depicted in Fig. 1, where the horizontal axis denotes the dimensionless superheat. The experimental data obtained by Nishio et al. [5] and Tokita and Djati [6] are illustrated by symbols. Note that Tokita and Djati's data are with respect to the heating surface of 1 mm height, where the smallest heat transfer characteristics was realized. In the experiment by Nishio et al., a polished aluminium surface whose total emissivity was assumed to be about 0.025 was utilized and so the contribution of radiation to the heat transfer coefficient may be sufficiently disregarded. On the other hand, Tokita and Djati used a brass heating surface whose total emissivity was 0.7 and thus radiation corrections to the obtained heat transfer data were necessary. The convective heat flux was determined by subtracting the radiative heat flux from the total heat flux and, using thus obtained convective heat flux, the Nusselt number was computed. The radiation heat flux q_{R} was evaluated from

$$q_{\text{R}} = \sigma\left(T_{\text{W}}^4 - T_{\text{S}}^4\right)/(1/\varepsilon + 1/\alpha - 1) \tag{40}$$

where ε is the total emissivity of a heating surface and α is the total absorptivity of water surface which was assumed to be unity. The present results indicate that, contrary to the previous theoretical results, C_0 is an increasing function of S_{P}. However, an increase rate of C_0 against S_{P} is weak, therefore, the theoretical result obtained by Nishio et al., i.e., $C_0 = 1.02$, is fairly reasonable. Detailed inspection of Fig. 1 reveals that our correlation approximates 91% of the available experimental data with an accuracy of less than $\pm 15\%$ and is more accurate than that derived by Nishio et al., which reproduces 86% of the plotted data with the same accuracy bounds.

4. Conclusions

The major conclusions that can be drawn from the present study are summarized as follows:

1. The dimensionless temperature profile needed for the analysis by boundary-layer integral methods should be determined so as to simultaneously satisfy the energy balance at the liquid–vapor interface and the energy equation.
2. C_0 defined by $Nu/X^{0.2}$ is an increasing function of the dimensionless superheat.
3. The heat transfer correlation given by Eqs. (37) and (39) reproduces the available experimental data with an acceptable accuracy.

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